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Pulse-code modulation for high-quality sound-signal distribution: Deviation, bandwidth and power for a frequency-modulated link

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RESEARCH DEPARTMENT

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PULSE-CODE MODULATION FOR HIGH-QUALITY SOUND-SIGNAL DISTRIBUTION: DEVIATION, BANDWIDTH AND POWER FOR A FREQUENCY-MODULATED LINK

Summary

A p.c.m. system for carrying thirteen high-quality sound signals is briefly described and reasons are given for choosing frequency-modulation for its transmission over a s.h.f. trunk link. The optimum frequency deviation is found through a consideration of the bandwidth required for the transmitted signal and the errors which will be caused by noise. It is shown that two distinct mechanisms are involved in the production of errors. The optimum characteristics of a practical system are calculated and the results are confirmed by experimental evidence.

1. Introduction

A pulse-code-modulation (p.c.m.) system[1] has been developed in BBC Research Department which converts up to thirteen high-quality sound programmes into a single train of binary digits and reconverts the binary signal into separate sound programmes without audible impairment. In the experimental coder and decoder, each of 13 sound signals are sampled at 38 kHz, each sample is quantised with 2^{14} possible levels (14 bits) and coded to form a 14-digit binary 'word'. The 13 'words' are read out serially, with two additional words necessary for synchronising and error detection, to form a complete 'frame' of words. The 'frames' occur at the sampling frequency and form a p.c.m. signal with a bit-rate of approximately 8 Megabits per second. Being a binary pulse-coded signal, it consists of pulses having two alternative voltage levels representing the digits '0' and '1' and the digit-rate is 8 MHz. The pulse shape chosen is a raised cosine, of the form $(1 + \cos \omega t)$ for $-\pi < \omega t < \pi$, lasting 0·25 μs, so that a succession of '0's or '1's gives a steady voltage and a transition from one to the other has the form of one half of the raised-cosine pulse. An alternating sequence of '0's and '1's would be a 4 MHz sinusoid but the base-bandwidth required to transmit random sequences with negligible waveform distortion extends to about 8 MHz (the bit-rate) although a high proportion of the power is confined in a somewhat smaller bandwidth. The bit-rate in the experimental equipment is $(13 + 2) \times 14 \times 38 \times 10^3 = 7{\cdot}98$ Mbit/s. In the operational equipment, it is proposed for a number of reasons to use a sampling frequency of 32 kHz and, since 13 bits per sample gives just adequate quantisation when so-called 'dither' signals are added,* a (13 + 2) channel system would require a frame of about 15 x 13 = 195 bits and a bit-rate of 6·24 Mbit/s. It is not however essential that the two 'words' for synchronising and error correction should contain the same number of digits as the programme words. By allowing 3 extra digits for synchronising etc., giving 198 bits per frame, the bit-rate may be equal to 6·336 Mbit/s which seems likely to become an accepted standard for trunk communication systems in the U.K.

Such p.c.m. signals occupy a greater bandwidth than that of all the sound signals they represent but they are considerably less prone to impairment by noise and interference. It has been proposed that this type of system should be used to convey high-quality sound programmes (including stereophonic pairs) along an s.h.f. trunk distribution system; a plan is at present in hand to use this system on a link between London and Holme Moss. Concerning the choice of a carrier-modulation system for the radio-frequency transmission of the p.c.m. sound signal, amplitude modulation (a.m.), frequency modulation (f.m.) and phase-shift keying (p.s.k.) would normally be considered. It can be shown that the use of suppressed-carrier a.m. (which may be regarded as a simple example of p.s.k.) would result in a somewhat lower bandwidth requirement than f.m. for the same peak carrier power and error rate although, to achieve this, it would demand good amplitude linearity. However, since present microwave link equipment uses f.m. for television and is readily available, it is expedient to choose the same form of carrier-modulation for the p.c.m. sound signal.

For frequency assignment purposes it is necessary to know the lowest value for the link bandwidth which allows the ruggedness of the p.c.m. system to be realised. The frequency deviation, r.f. filtering, carrier-to-noise ratio and digit error rate are all interdependent but a start may be made by considering the r.f. bandwidth required to contain 99% of the transmitted signal power as a function of the frequency deviation. Having deduced this relationship, the dependence of error-rate on carrier-to-noise ratio and deviation may be calculated and an optimum value found, as will be shown. Confirmatory measurements of the spectrum of a binary f.m. signal have been made and are discussed in Section 5.

* These effectively double the number of levels (see Reference 2).

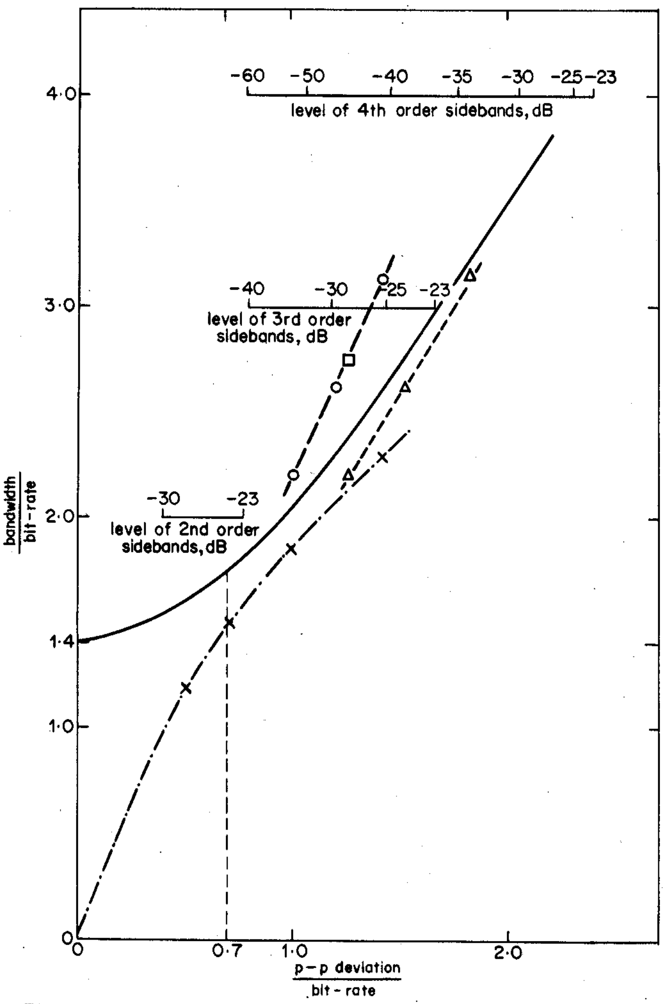


Fig. 1 - Bandwidth required for a pulsed f.m. signal

-30 Level, dB, of sideband for sinusoidal f.m. at a frequency equal to half the bit-rate

Bandwidth containing 99% signal power

□ for frequency modulation by random noise (Section 2)

o——o for a random sequence of rectangular pulses (Ref. 2)

Δ- - - -Δ for a random sequence of raised-cosine pulses deduced from Ref. 2

x— - —x for a random sequence derived from spectra of Fig. 7(*a*)

—— Estimated bandwidth for a practical p.c.m. system giving effective reception and containing at least 99% of the signal power

2. Bandwidth and deviation

The spectrum of a signal which is frequency-modulated by a random sequence of binary raised-cosine pulses is not easy to calculate but a result has been published for rectangular pulses[3] and the result for raised-cosine pulses may be estimated from this; the bandwidth required for both types of pulse is shown in Fig. 1, plotted against peak-to-peak deviation and normalised to the bit rate (dashed curves). Also shown (by means of horizontal scales) are the levels of the discrete sidebands produced by an alternating pulse sequence at half the bit-rate, because such sequences may be used in practice as mentioned in Section 5.1; when each of these sidebands is 23 dB below the carrier level, the pair of them carry 1% of the power. An additional point is given representing deviation by random noise band-limited to the bit-rate, with a r.m.s. value 9 dB below the peak-to-peak deviation shown.

A further series of results has been obtained from measurements of the 99% power bandwidth for a pseudo-random sequence of binary raised-cosine pulses for four different values of deviation (shown by crosses in Fig. 1): these measurements are described in Section 5.1. The broken line drawn through the experimental points has been extended to the origin, giving a lower bound to the bandwidth required to accommodate 99% of the power. This result is, however, somewhat artificial near the origin since, at very low deviation, the power in the carrier-frequency component alone will approach 99%. For effective transmission the bandwidth must also be sufficient to carry the modulation with negligible distortion; it must therefore tend, at very low deviation, to be twice the base-bandwidth in order to include the first-order sidebands at the highest base-band frequency which is, ideally, greater than the bit-rate itself for a raised-cosine pulse. But, if we assume that the base-band spectrum may be limited to contain 99% of the pulse power, the permissible cut-off frequency is approximately 0·7 times the bit-rate, as shown in Fig. 2. Restriction of the binary-signal bandwidth to 0·7 times the bit rate will inevitably lead to some pulse distortion and reduce the noise-immunity of the system but this impairment has been shown to be small and it will be discussed later.

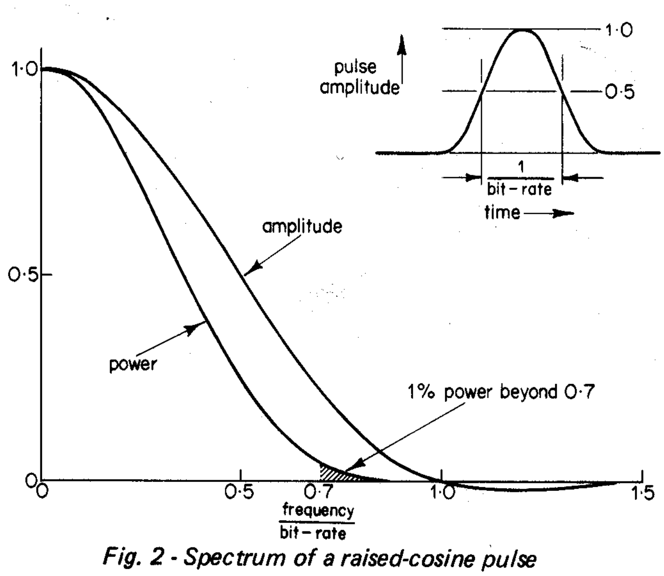


Fig. 2 - Spectrum of a raised-cosine pulse

The lowest permissible r.f. bandwidth at very low deviation is therefore 1·4 times the bit rate; at higher deviations, the bandwidth must include 99% of the transmitted power. These requirements are met by the bold curve drawn on Fig. 1 which gives the best deduction from the available evidence. If we now assume that the receiving terminal will restrict the r.f. bandwidth to the values given by this curve and will restrict the base-bandwidth to 0·7 times the bit rate, we may derive a relationship between the deviation and the noise power in the receiver. The optimum deviation will be that for which the noise causes the least error in the decoded base-band signal.

3. Noise and errors

3.1. Noise in a digital base-band system

An arbitrary sequence of regularly-timed raised-cosine pulses will have a waveform which, in a repetitive display, lies wholly between two well-defined outer levels, with an unfilled region bounded by transitions between these levels in the shape of an 'eye' centred on each epoch. In the p.c.m. decoder the signal is regenerated by sampling the waveform with short pulses which are synchronised with the digit-rate, producing either a '0' or a '1' depending on whether the signal passes below or above the centre of the eye. The 'eye-height' is ideally equal to the vertical spacing between the two extreme levels but, in the presence of noise, the eye-height will be reduced. Each time the waveform passes the wrong side of the centre of the eye, a digit error will result. Pulse distortion, which will arise if the signal suffers phase distortion in transmission, and other instrumental inaccuracies will also reduce the effective eye-height so that more errors will occur for a given noise level. For example, the restriction of the pulse spectrum to 0·7 times the digit rate has been shown to reduce the eye-height to about 95% and this is equivalent to an increase of 0·5 dB in the noise level as far as the consequent increase in error rate is concerned.

The importance of a given error rate depends on the design of the decoder. For example, the presence of an error may be detected in the decoder by various forms of parity check, in which certain selected digits are summed in the coder and a separate digit is transmitted as a check of the oddness or evenness of the sum. Having detected an error, it can be largely concealed in the case of sound signals by substituting a repeat of the previous sample. Using fairly simple techniques, a digit error rate of about 10^{-6} gives an imperceptible impairment (Grade 1)* and an error rate of 10^{-3} is definitely objectionable (Grade 5).*[4] More elaborate techniques are available[4] which may allow the corresponding error rate to rise to 10^{-3} for the limit of perception or 3×10^{-2} for the limit of acceptability, but it is doubtful if the adoption of such techniques is justified in the present application.

3.2. Noise in a digital f.m. system

If we now consider a frequency-modulated r.f. carrier having a fixed amplitude, it is clear that a reduction of the deviation will reduce the amplitude of the base-band signal, reduce the base-band signal-to-noise ratio and increase the probability that the noise voltage will cause an error in decoding the signal. On the other hand, if the deviation is increased, the r.f. bandwidth required to accommodate the spectral power must be increased, as shown by Figure 1, giving an increase in the noise power in the r.f. band and a greater probability that the noise voltage will exceed the peak carrier amplitude. When this happens, the phase of the resultant signal may suffer a transition of 2π radian,[5] causing a pulse in the demodulator output which may cause a digit error in the decoder.

* These grades correspond to those of the EBU six-point impairment scale (CCIR Report 405, Oslo 1966, Note 2).

The actual mechanism, particularly for intermediate values of deviation, is a complex process. Nevertheless it is convenient for obtaining an approximate solution to assume that there are two distinct mechanisms by which errors may be caused which we may call modulation-failure at low deviation and carrier-failure at high deviation. At some intermediate, optimum value of deviation there will be a minimum error-rate for a given carrier power. Figure 3 shows contours of constant error rate as graphs relating the peak-to-peak frequency deviation to the carrier power for a 6·3 Mbit/s system. The derivation of these contours is given in Sections 3.3 and 3.4 and a comparison between the theoretical and measured results is given in Section 5.2.

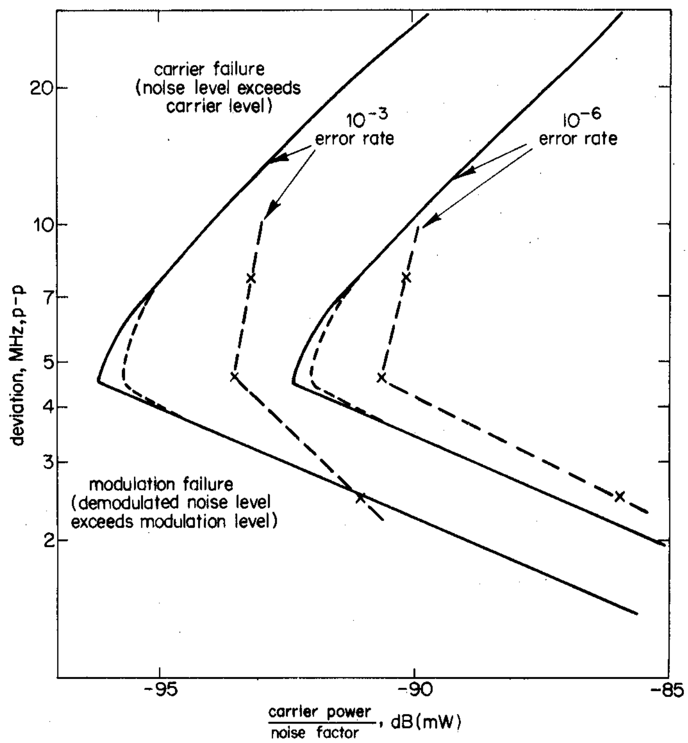


Fig. 3 - Error probability contours for a 6·336 Mbit/s f.m. system

——— Theoretical contours for each mechanism
- - - - - Theoretical contours for both mechanisms
x— — —x Measured contours for a practical system

At the optimum deviation, which is independent of the carrier power, the two failure mechanisms will contribute about equally to the production of errors. But they will not contribute equally to the base-band noise power because they give rise to two types of noise which have different spectra and which depend on the carrier-to-noise ratio in different ways; a discussion is given in an Appendix (Section 10).

Before describing the two failure mechanisms in detail, it is helpful to refer the vector diagrams of Figure 4. Taking, as a reference, the phase of a stationary signal at the centre-frequency f_0 of the r.f. pass-band, the peak voltage of the frequency modulated carrier is represented by a vector whose end point moves on the arc of a circle. Its

phase angle, which is the integral of the modulated carrier frequency, is stationary at an extreme phase only, at the instants when the frequency is passing through f_o during a transition in either direction between a '0' and a '1'. The smallest peak-to-peak phase deviation therefore occurs when the signal is a series of alternating '0' and '1's so that the frequency deviation alternates between $\pm\frac{1}{2}D$ at a frequency of $\frac{1}{2}f_b$, giving a peak-to-peak phase deviation of $2D/f_b$ radians where D is the peak-to-peak frequency-deviation and f_b is the bit rate. For a succession of '0's or '1's, the phase will progressively decrease or increase with an angular velocity of πD radians/sec.

The noise is represented by a circle, at the head of the carrier vector, whose radius is, in Figure 4, equal to the amplitude v_n of a notional single-frequency noise vector having the same total power as the noise. If the r.m.s. carrier-to-noise ratio is s, the length of the vector representing the amplitude of the carrier, will be s times the radius of the circle. The r.m.s. phase deviation of the carrier due to noise will be $1/(s\sqrt{2})$ radians when the noise power is relatively low. The noise vector itself may be considered as the sum of two orthogonal components along Cartesian axes, uncorrelated with each other and having a Gaussian distribution of amplitude. Figure 5 gives the statistics of the component distribution by relating the proportion of the time, p, for which the instantaneous noise excursion exceeds a certain level (both positive and negative peaks) to the factor, μ, by which that level exceeds the r.m.s. value of the noise component. If peaks having one particular polarity are considered, the time proportion will be p/2.

In order to avoid a rigorous mathematical treatment the following Sections will introduce some assumptions which can be justified with the aid of Figure 4 for deriving an approximate solution.

3.3. Carrier failure

Referring to Figure 4, the locus of a noise voltage peak is shown which exceeds the amplitude of the carrier vector. The proportion of the time, p, for which the component of noise parallel to the carrier vector is greater than the carrier is given by putting $\mu = s\sqrt{2}$ in Figure 5. For half of this time, p/2, the peak of the noise locus will be not only greater in amplitude but opposite in phase to the carrier vector and therefore fall below the horizontal line through the origin in Figure 4. We shall assume that, when this occurs, the resultant vector locus will enclose the origin with a large angular velocity. This will introduce a step of 2π radians, leading to a large false pulse in the demodulated digital signal. We shall also assume that, when this occurs, a digit will be misread by the decoder. The assumption that errors will be caused for all of the time p/2 may be justified by the following observations. Although, for a carrier at the reference mid-band frequency, f_o, the noise peak locus may miss the origin, owing to a simultaneous noise peak in quadrature with the carrier, the carrier frequency will be at $f_o \pm \frac{1}{2}D$ at the crucial digit sampling instants so that the angular velocity of the carrier vector will be a maximum and ensure that it passes through a phase opposite to that of the noise peak being considered. Moreover the resultant phase-step caused by the noise peak will, most probably, be of the opposite sign to the signal phase transition and therefore cause an error to digits of either polarity. For a digit error rate of 10^{-6} the value of μ for $p = 2{\cdot}10^{-6}$ is $\mu = 4{\cdot}75$ or 13·5 dB and s is 3·35 or

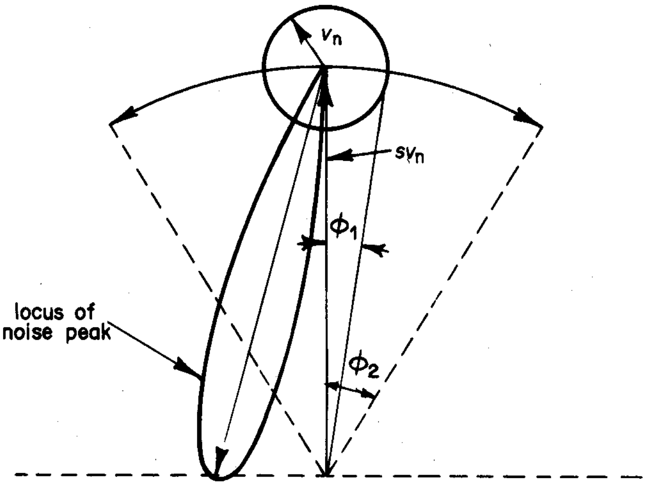


Fig. 4 - Vector diagram for a noise f.m. signal

$v_n = \sqrt{2}$ x r.m.s. noise voltage
s = r.m.s. carrier-to-noise ratio
$\phi_1 = \dfrac{1}{s\sqrt{2}}$ radian r.m.s. for large s
$\phi_2 = \dfrac{D}{f_b}$ radian peak for 010101 . . . signal
D = frequency deviation, MHz peak-to-peak
f_b = bit-rate, MHz

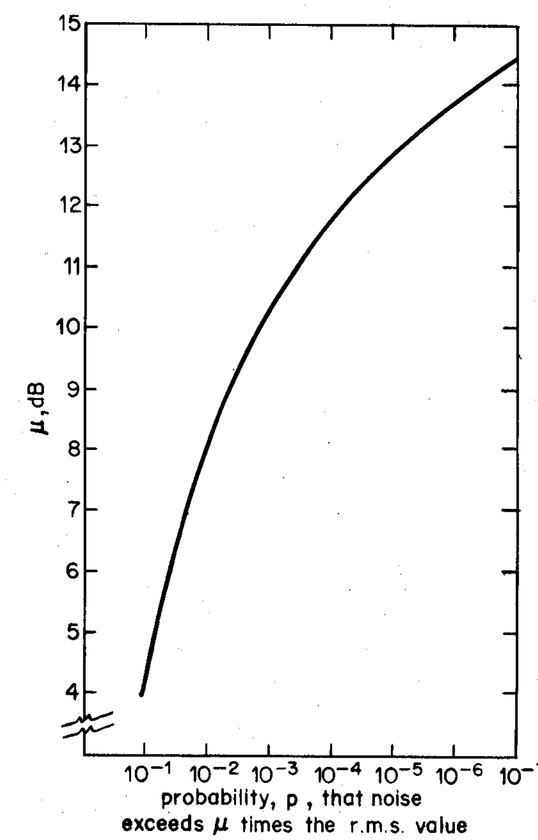


Fig. 5 - Gaussian amplitude statistics

10·5 dB. For an error rate of 10^{-3} the appropriate value of μ is 3·05 or 9·7 dB and s is 2·15 or 6·7 dB. The carrier power is

$$P_c = s^2 P_o N B \qquad (1)$$

where P_o is the noise power per MHz at 290°K (– 114 dBmW), N is the noise factor and B is the bandwidth given by Figure 1 as a function of the deviation and the bit rate. Substitution gives the upper parts of the contours in Figure 3.

3.4. Modulation failure

Components of noise which are in quadrature with the carrier will produce a phase disturbance having a r.m.s. phase deviation of $1/s\sqrt{2}$ radians and containing components with a flat spectrum up to $\tfrac{1}{2}B$. After frequency-demodulation, in which the resultant (carrier + noise) phase is differentiated, the noise power spectrum will be parabolic and extend up to a frequency $f_{max} \leqslant \tfrac{1}{2}B$ determined by a base-band filter as shown in Figure 6. If we take, for reference, a signal modulation which is sinusoidal at $\tfrac{1}{2}f_b$, corresponding to an alternating series of '0's and '1's and having a peak-to-peak frequency deviation D, the base-band signal-to-noise ratio is

$$S = \frac{\text{p – p base-band signal}}{\text{r.m.s. base-band noise}}$$

$$= \frac{D}{\tfrac{1}{2}f_b} \cdot s\sqrt{2} \cdot \sqrt{\frac{\tfrac{1}{2}B}{f_{max}}} \cdot \frac{\tfrac{1}{2}f_b}{f_{max}} \cdot \sqrt{3} \qquad (2)$$

where the first term in the product is the peak-to-peak phase deviation of the signal, the second and third terms give the reciprocal of the band-limited r.m.s. noise (in terms of phase excursion) and the last two terms correct the r.m.s. noise to allow for the parabolic base-band power spectrum.

When the peak noise exceeds half the peak-to-peak signal in either polarity the probability of a digit error will be halved since half the noise peaks will be in phase with the signal. So we put $S = 2\mu$ where μ is the value for p equal to twice the digit-error rate. Re-arranging Equations 1 and 2, the carrier power is

$$P_c = \frac{\mu^2}{D^2} \cdot \frac{4}{3} \cdot (f_{max})^3 \cdot P_o \cdot N \qquad (3)$$

where, as before, μ = 4·75 or 13·5 dB for a 10^{-6} digit error rate and μ = 3·09 or 9·7 dB for a 10^{-3} digit error rate.

If the base-band filter cuts off at $f_{max} = f_b$, then the p.c.m. pulse shape will be preserved and no interdigit confusion can occur. However, as suggested in Section 2, a reduction of f_{max} of from f_b to $0{\cdot}7 f_b$ would include 99% of the pulse energy; the consequent distortion would have the equivalent effect of reducing the carrier-to-noise ratio by about 0·5 dB (Section 3.1) whereas the bandwidth reduction would reduce the triangulated baseband r.m.s. noise by 4·5 dB, giving a worthwhile improvement of 4 dB. Taking f_{max} equal to $0{\cdot}7 f_b$, Equation 3 leads to the lower parts of the contours in Figure 3.

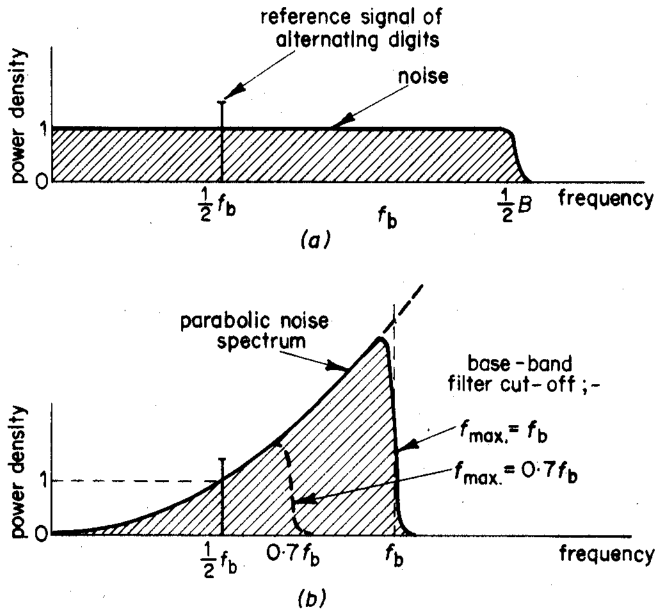


Fig. 6 - Base-band spectrum of noise causing modulation failure

(*a*) Power output of a notional phase detector

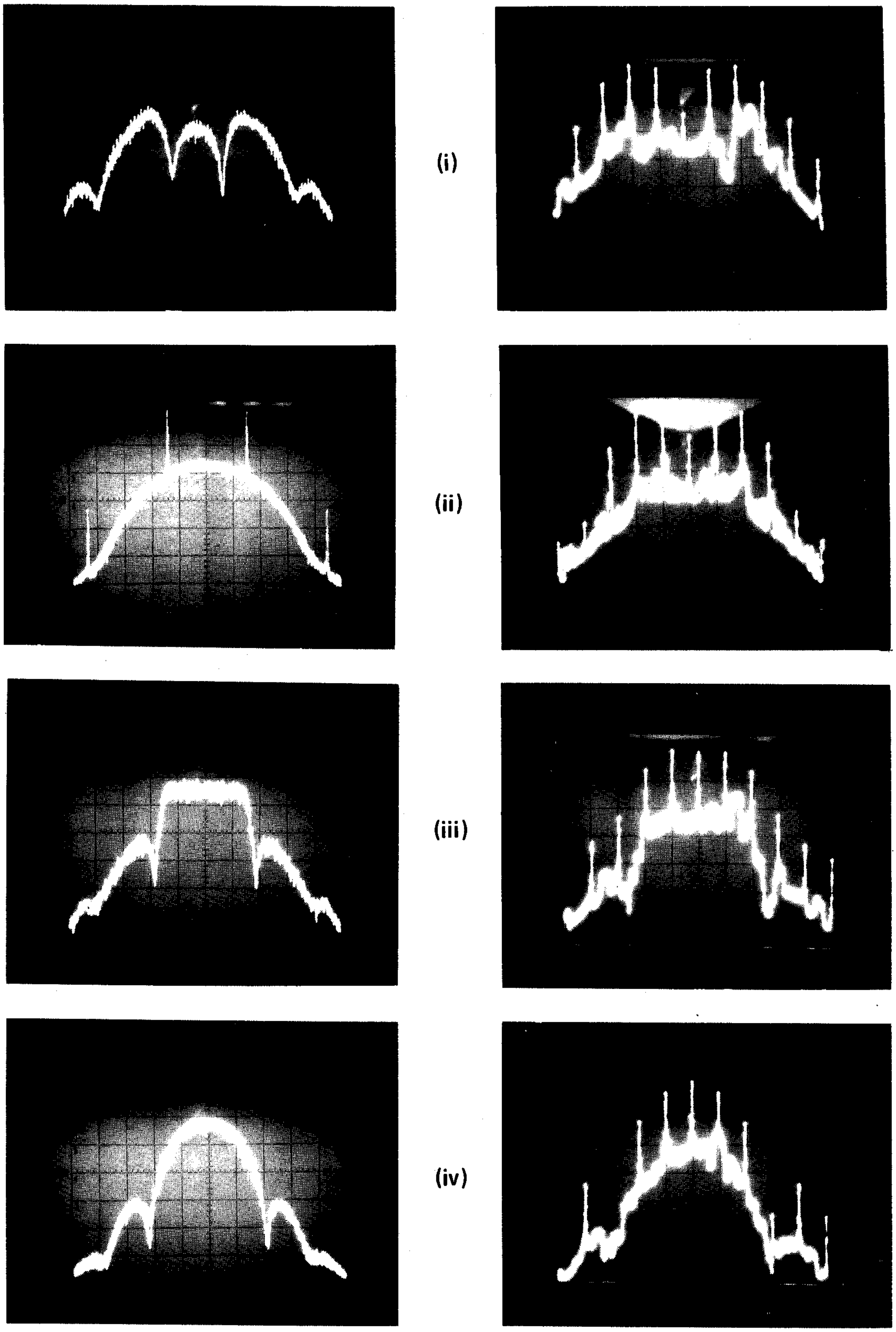
(*b*) Base-band power in output of f.m. discriminator

4. Optimum deviation and bandwidth

The contours of Figure 3 are shown with sharp cusps at a deviation of 4·5 MHz peak-to-peak, representing the deviation requiring the lowest carrier power for a given error rate. The cusps will not be so sharp in practice as there will be, at this optimum deviation, some coexistence of errors caused by carrier failure and modulation failure, but this will make only a fraction of a dB increase in the optimum carrier power: this is indicated by the broken parts of the contours in Figure 3.

It will be noted that the optimum deviation is $0{\cdot}7 f_b$ which is the same as the value chosen for the cut-off frequency, f_{max}, of the base-band filter; although these two quantities are interdependent, their apparent identity is fortuitous.

Referring to Figure 1, in order to transmit a 6·3 Mbit/s signal with the optimum deviation of $0{\cdot}7 f_b$, the system bandwidth must be about 11·5 MHz. The transmitter bandwidth assignment must be greater than this to allow for a drift in carrier frequency and to minimise interference and distortion which may be caused either by radiating or attenuating the discrete sidebands arising from an alternation of digits in the p.c.m. signal. As described in an Appendix, Section 10.2, if the second-order sidebands at ±6·3 MHz are removed by a 11·5 MHz filter at the transmitter output, then a significant amount of amplitude modulation and waveform distortion will result. And if the r.f. filter precedes the output stage of the transmitter, nonlinearities will tend to reintroduce these sidebands. It is therefore necessary to allow a transmission bandwidth of at least 12·6 MHz. A bandwidth assignment of 14 MHz will be adequate and allow a tolerance of ±0·7 MHz in the nominal mean carrier frequency. The discrete third-order



(a) *(b)*

Fig. 7 - R.F. spectra

(*a*) Pseudo-random signal at 5·8 Mbit/s (*b*) 13-channel p.c.m. signal at 8 Mbit/s with dummy data in 12 channels

$$\frac{\text{peak-to-peak deviation}}{\text{bit-rate}} = \begin{cases} \text{(i)} & 1{\cdot}4 \\ \text{(ii)} & 1{\cdot}0 \\ \text{(iii)} & 0{\cdot}7 \\ \text{(iv)} & 0{\cdot}5 \end{cases}$$

sidebands at ±9·5 MHz, which will be at least 40 dB below the carrier level, may be reduced to a level of −60 dB by a filter which attenuates components outside a bandwidth of say 18 MHz by about 20 dB.

The i.f. bandwidth of the receiver must be kept fairly close to the minimum value of 11·5 MHz in order to preserve the carrier-to-noise ratio but it may be as high as 12·6 MHz to avoid excessive waveform distortion, as discussed in the Appendix, Section 10.2.

5. Experimental work and discussion

5.1. spectral measurement

Figure 7 shows experimental results for the spectrum of a 70 MHz signal when it is frequency-modulated by a binary, raised-cosine pulse stream to give a peak-to-peak deviation of +3 dB, 0 dB, −3 dB and −6 dB with respect to the bit rate. Two spectra are shown for each value of normalised deviation; the modulation for the left-hand spectra, Figure 7(*a*), was a pseudo-random sequence of pulses at 5·8 Mbit/s and that for the right-hand spectra, Figure 7(*b*), was an 8 Mbit/s signal from the experimental 13-channel equipment in which only one channel carried a coded sound signal and the remainder had dummy pulses in alternating pairs with a fundamental frequency of 2 MHz, one quarter of the bit rate. The reference level was, in all cases, 0 dB for the unmodulated carrier.

The spectra of Figure 7(*a*), for the pseudo-random sequence, show an interesting progression in the distribution of energy. The bandwidth containing 99% of the energy has been calculated from these results and is shown in Figure 1; it increases as the frequency deviation increases but in a non-linear fashion, owing to the presence of notches in the spectrum. The frequency separation of these notches varies inversely as the deviation but they vanish when the deviation is exactly equal to the bit rate and are replaced by discrete spectral lines, separated by the bit frequency. These spectral lines carry one quarter of the energy in the signal and they occur because, at this particular deviation, the successive contributions at the frequencies corresponding to '0' and '1' all add in the same phase, even for a random message.[3]

Turning to Figure 7(*b*), the same features are visible at the same relative deviation but they are masked to some extent by the coherence of the dummy data, producing lines every 2 MHz.

A deviation equal to the bit rate is to be avoided because the strong spectral lines may present a source of interference between co-channel links; the choice of 0·7 times the bit rate, which has been shown to require the least carrier power in the presence of noise, is particularly fortunate in its spectral distribution in having, near to the carrier frequency, a compact 'pedestal' of energy standing 20 dB clear of any other components.

In a practical system, there will be quiet periods in all sound channels. This would tend to produce a binary signal containing a preponderence of '0's or '1's which would embarrass the restoration of the d.c. value of the signal both at the modulator and at the decoder. In order to avoid this situation, it is proposed to change the polarity of alternate digits in the transmitted bit-stream. This would result in a higher degree of coherence in the r.f. signal during programme pauses, leading to spectral lines at multiples of half the bit rate. The second-order lines will, however, be well within the overall r.f. bandwidth chosen in Section 4.

5.2. Error-rate measurement

Figure 3 shows, in addition to the theoretical error-rate contours for a 6·3 Mbit/s system, the results of measuring the error-rate caused by adding random noise in the 8 Mbit/s system after appropriate re-normalising to the bit-rate.*

Each of three ten-section Butterworth band-pass filters of different bandwidths were placed in turn in the r.f. signal path and the deviation adjusted according to the solid curve of Figure 1 for each filter. The noise level was then varied, the error rate recorded and the carrier-to-noise ratio for each error-rate contour was found by interpolation. The demodulated, base-band, signal was passed through a low-pass filter with a cut-off frequency of 5·6 MHz, or 0·7 times the bit-rate, as suggested in Section 2 and assumed in the subsequent calculations.

The effect of the base-band filter was to reduce the eye-height of the pulse-stream to 95% as has been mentioned. But a further reduction of eye-height was caused by the conditions of the test, which involved long cable-links between the source of the test-signal and the laboratory. These were shown, on a base-band loop test, to reduce the eye-height to an average of 75% and a minimum of 60% for the worst combination of digits. We may assume that, if these defects had been removed from the base-band path, and the eye-height were restored from 75% to 95%, the carrier power could be reduced by 2 dB.

Making this allowance for instrumental defects, the measurements tend to confirm the theory in respect of both the optimum deviation and the carrier power required for a given error rate.

6. Fading and co-channel interference

If we assume a receiver noise factor of 12 dB and use the optimum deviation of 4·5 MHz peak-to-peak for a 6·336 Mbit/s system, Figure 3 shows that the received carrier power may be allowed to fall to −80 dB(mW) before impairment is caused to the sound signals. When planning a p.c.m. s.h.f. link, the allowance made for fading of the signal below the free-space transmission level must take into account the fact that the signal will be virtually unusable at a received carrier power of −84 dB(mW).

* These measurements were made by W. Murray.

If we consider co-channel interference, initially in the absence of noise, a c.w. signal close to the mean carrier frequency will produce errors through carrier failure as the interfering-to-wanted-carrier ratio rises to 0 dB. But, at greater frequency offsets, the onset of modulation-failure will occur as the rate of change of phase of the resultant drops to zero at the digit epochs, so causing the binary signal to cross the centre of the 'eye'. This will occur at a carrier ratio of −6 dB when the offset is equal to 0·7 times the bit rate of twice the peak-to-mean deviation. At still higher offsets, the impairment is likely to diminish as the base-band filter limits the rise-time of the transitions. We may therefore take a figure of 6 dB as the required protection ratio for the chosen p.c.m. transmission against co-channel c.w. or f.m. interference in the absence of noise.

But co-channel interference at levels below −6 dB will have 'used up' some of the available eye-height and therefore render the system more prone to impairment by noise. Figure 8 shows the results of calculation giving the wanted carrier level for two error rates as a function of the interfering carrier level.

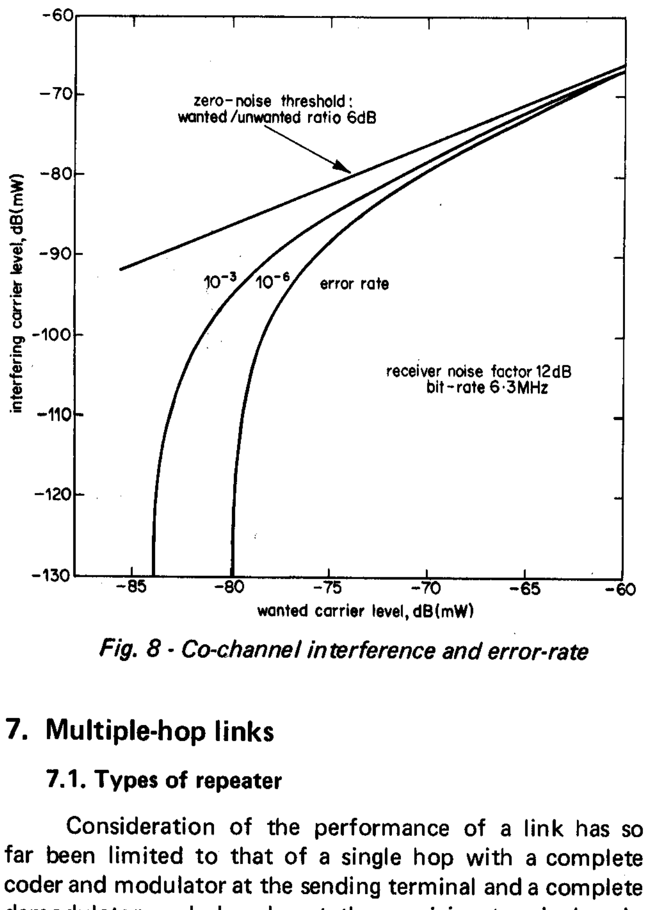


Fig. 8 - Co-channel interference and error-rate

7. Multiple-hop links

7.1. Types of repeater

Consideration of the performance of a link has so far been limited to that of a single hop with a complete coder and modulator at the sending terminal and a complete demodulator and decoder at the receiving terminal. In practice, there will be several such links in tandem; each intermediate terminal equipment will receive an attenuated signal, convert it to another channel frequency and retransmit it at the highest permitted power level. But there is a choice to be made concerning the treatment of the signal within each repeater; there are three main alternatives which have different characteristics in the presence of random noise.

First, each repeater in a multi-hop link may contain a complete demodulator and modulator (regenerative repeater) so that each hop is, in itself, complete with a base-band input and output. At the r.f. input of any such repeater, the carrier-to-noise ratio will be that determined on only the previous hop but the signal will contain errors accumulated from earlier hops. Assuming that all n hops in the link are equal in attenuation, the overall error rate will be proportional to n, as shown in curves (*a*) (*b*) and (*c*) of Figure 9.

Second, the repeater may simply amplify the signal to the required power level and re-transmit it together with any received noise (simple repeater). In this case, the final terminal of a uniform n-hop link will receive n times the noise power received in a single hop. The final r.f. carrier-to-noise ratio will therefore be proportional to $1/n$ for a given hop-attenuation. The final error rate can be restored to that of a single-hop link only by reducing the attenuation per hop (or increasing the carrier power) by a factor n, as shown in curves (*a*) (*f*) and (*g*) of Figure 9.

Third, the simple repeater, just described, may be fitted with an efficient r.f. amplitude-limiter (limiting repeater) so that those components of noise which are in phase with the carrier are not passed on from hop to hop. This will remove the cause of about half the errors, assuming that the frequency-deviation chosen is optimal for a single-hop link, so that the number of errors will be half that of the link using simple repeaters, as shown in curves (*a*) (*d*) and (*e*) of Figure 9.

The advantage of using only a limiter is apparently small as it allows a saving in signal power of less than 1 dB for a given error-rate. Nevertheless, a much greater advantage is possible in certain conditions discussed below. In any case, it would be unusual for a simple f.m. repeater to be perfectly linear in amplitude; all practical non-regenerative f.m. repeaters may be regarded as limiting repeaters.

7.2. Optimum deviation for links with limiting repeaters

It should be noted that, under somewhat idealised conditions in which the link contains several hops of equal attenuation and uses amplitude-limiting repeaters, errors caused by the mechanism of carrier-failure are far less probable than those caused by modulation failure. This means that the frequency deviation which is optimum for a single hop link is no longer optimum for an n-hop link, and there would be an advantage if it could be increased by a factor approaching n so as to reduce the probability of modulation failure to the lower value for the probability of carrier failure. The improvement to be gained may be seen from a comparison of the signal powers required in a multiple hop link with and without an increase in deviation to the optimum* value for the number of hops, as shown in Table 1 calculated for a 6·3 Mbit/s system.

* 'optimum' deviation here means the deviation permitting the minimum transmitter power for a specified error rate. In practice, a lower deviation might be used to conserve spectrum space at the expense of increased transmitter power.

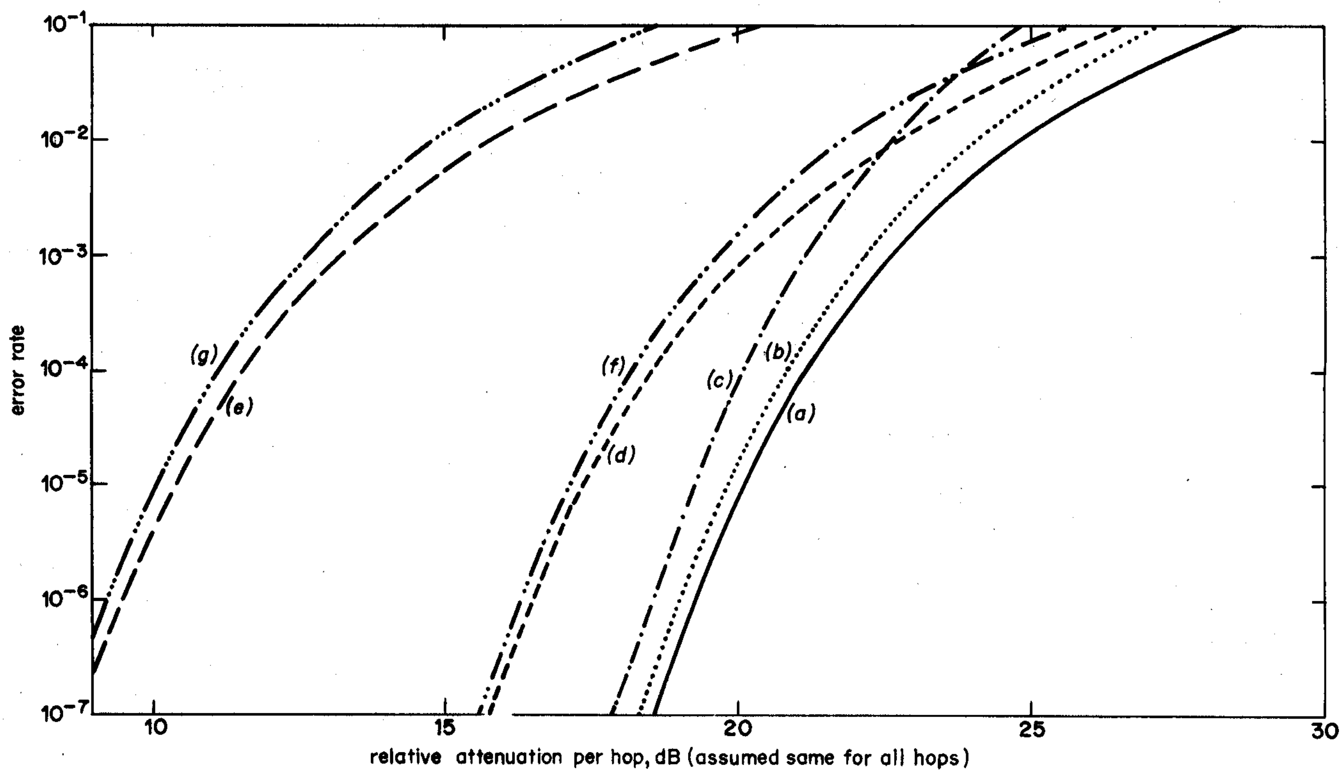


Fig. 9 - Error rate for multi-hop links with uniform attenuation

(*a*) ———— Single hop

(*b*) •••••••••• Regenerative repeaters, 2-hop (*d*) - - - - - - Limiting repeaters, 2-hop

(*c*) •—•— Regenerative repeaters, 10-hop (*e*) — — — Limiting repeaters, 10-hop

(*f*) ••——••—— Simple repeaters, 2-hop

(*g*) —•••—— Simple repeaters, 10-hop

TABLE 1

Optimum deviation for link of several equal hops, using amplitude-limiting repeaters

No. of hops	Optimum p–p deviation MHz	Channel bandwidth required for optimum deviation, MHz	Signal power required for same error rate, dB	
			Deviation optimum for single hop	Deviation optimum for multiple hop
1	4·5	14	0	0
2	5·8	15	3	0·7
4	7·5	17	6	1·4
10	10·3	19	10	2·7

In practice, however, the attenuation will be the same for each hop of a link only when it is either close to the free-space value or is caused by a generally high absorption of the atmosphere owing to humidity or precipitation over the whole link. This is unlikely at frequencies of 7 GHz and lower, though it may predominate at frequencies near to 20 GHz.

7.3. Fading in multiple-hop links

The principal cause of fading signals at frequencies below 10 GHz is a non-uniformity in the refractive index of the atmosphere and the attenuation is found to follow a Rayleigh distribution with fair accuracy. The general level of attenuation may depend on weather conditions, but the

occurrence of deep fades at say, 7 GHz, is a sporadic local phenomenon affecting only one hop of a link at a time, and has negligible correlation with fades on other hops. This means that the overall probability of a fade deeper than a set margin, or the 'outage' time-proportion for a link, will tend to be proportional to the number of hops and will tend to be independent of the type of repeater, because nearly all the errors will be determined in the hop suffering the most attenuation at the time and will not be affected by subsequent limiters or regenerators. The error rate will therefore be proportional to the number of hops for all types of repeater, and will follow curves (*a*) (*b*) and (*c*) of Figure 9.

In these circumstances, an economical n-hop, f.m. p.c.m. link at 7 GHz, with typical path lengths for each hop, will employ non-regenerative repeaters, the frequency deviation will be the optimum for a single hop and the carrier power will be sufficient to give a hop-outage time of $1/n$ of the desired overall outage time.

8. Conclusions

The main features of a prototype 13-channel, binary p.c.m. sound system, operating at 8 Mbit/s, have been described elsewhere.[1] In the application of such a system for internal distribution of BBC programmes by means of a s.h.f. trunk link, operating at 6·336 Mbit/s, the r.f. spectrum has been deduced theoretically (Figure 1) and an optimum frequency-deviation, requiring the least carrier power, has been indicated and supported by the results of measurement (Figures 3 and 7), leading to a choice of 4·5 MHz peak-to-peak deviation for the 6·336 Mbit/s signal and requiring a total r.f. bandwidth allocation of 14 MHz (Section 4). The received carrier power on a single-hop link may, in this optimum condition, be allowed to fade to −80 dB(mW) before the sound signals become impaired by noise alone, assuming a receiver noise factor of 12 dB as shown in Figure 3. The additional effect of co-channel interference has been calculated and is given in Figure 8.

The discussion given (Section 7) of the requirements of a practical 7 GHz multiple-hop link when any deep fading is uncorrelated between hops, leads to the conclusion that simple, non-regenerative repeaters will be adequate and that the optimum frequency deviation will be the same as for a single-hop link. A small increase is required in the carrier power for each hop over the −80 dB(mW) level mentioned above, amounting to about 1 dB increase for a 10-hop link.

9. References

1. Pulse-code modulation for high-quality sound-signal distribution: instrumentation of experimental multiplex system. BBC Research Department Report No. 1970/36.

2. Pulse-code modulation for high-quality sound-signal distribution: quantising distortion at very low signal levels. BBC Research Department Report No. 1970/18.

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5. MAURICE, R.D.A. 1957. V.H.F. broadcasting: reduction of impulsive interference in f.m. reception. *Elect. & Radio Engr.*, 1957, **34**, 8, pp. 300 – 309.

10. Appendix

10.1. Base-band noise spectrum

Figure 10 shows the base-band noise spectrum for different error rates in a system with optimum frequency deviation. At low error rates, the r.m.s. noise tends to be proportional to the frequency below the cut-off frequency of the base-band filter f_{max}. This is because most of the power arises from the phase modulation of the carrier by noise and this has a normal distribution of amplitude, a triangular spectrum after demodulation and a power that is inversely proportional to the carrier-to-noise ratio. At high error rates, the spectrum tends to be uniform because the errors caused by 100% amplitude-modulation of the carrier (carrier-failure) emerge from the demodulation as impulses which have a non-normal amplitude distribution, a uniform spectrum and a power which is proportional to their number. Although the errors are equally probable from either mechanism, they increase at a much more rapid rate than the corresponding decrease in carrier-to-noise ratio, so that, as the error rate increases, the spectrum tends to change from that associated with modulation failure to that associated with carrier failure, that is, it changes from triangular to uniform.

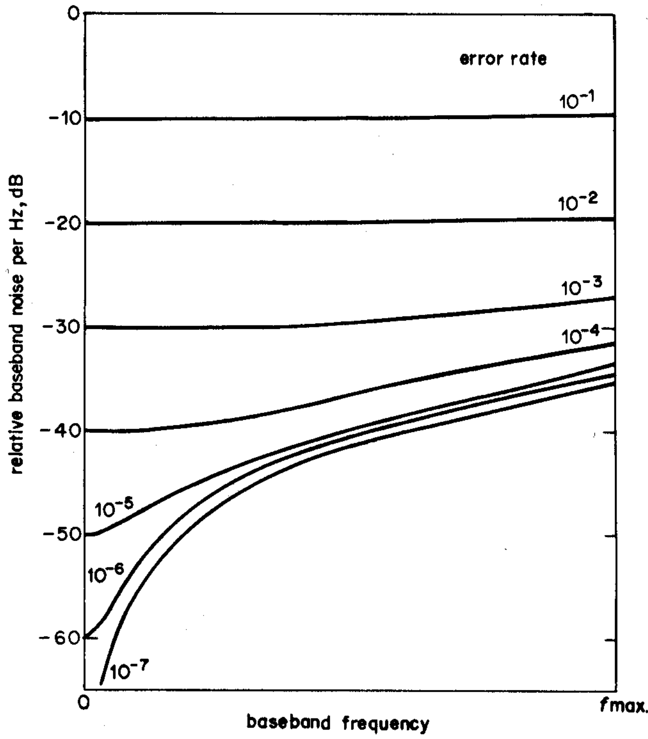

Fig. 10 - Base-band noise spectrum and error rate

10.2. Waveform distortion in a f.m. system

The signal will suffer some distortion in an optimised system, mainly through the effects of frequency-band limiting; the distortion may take two forms. First, there will be a non-uniformity of the phase/frequency characteristic of the filters in the signal path; this may be corrected by equalisers at appropriate stages. Second, there will be some phase and amplitude distortion caused by the loss of some signal sidebands in the same filters. This cannot be corrected so simply. For example, an alternating series of digits will cause a sinusoidal frequency-modulation at half the bit-rate, $\frac{1}{2}f_b$; if the second-order r.f. sidebands at $\pm f_b$ are passed by the r.f. filter but the third order sidebands at $\pm 1{\cdot}5\ f_b$ are removed, as suggested in Section 4, the base-band waveform will suffer about 1·5% distortion. If, on the other hand, the r.f. filter were reduced in bandwidth so as to remove the second-order sidebands, about 11% a.m. would be generated; this would be removed in the amplitude-limiter, leaving about 3% waveform distortion, which would decrease the noise-immunity of the system by about 1 dB. Efficient amplitude-limiting in the receiver, particularly at low carrier-to-noise ratios, is of course essential for the realisation of the theoretical noise performance of a f.m. system.